

# Application of Nonlinear Dynamics in Analysis of Electroencephalograph

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**ABSTRACT:** Nonlinear dynamics has been used to analyse Electroencephalograph (EEG) time series, in recent years, which has opened new possibilities to the dynamic knowledge of the brain. In this paper, we review researches on the treatment of EEG (sleep stage, anesthesia, process of cognition, schizophrenia, dementia and epilepsy) using nonlinear dynamics methods, and try to better understand brain neurodynamics.

**Key words:** EEG; Nonlinear dynamics; Sleep stage; Anesthesia; Process of cognition; Schizophrenia; Dementia; epilepsy

## 1 Introduction

Before the application of nonlinear dynamics, EEG series were analyzed mainly in two fields: the time domain and the frequency domain. These traditional methods often view EEGs as stationary and definite signals. In the recent years, nonlinear dynamics theory, which has been developed since the 1960's and viewed as the third revolution of nature science in the twentieth century, has been developing fast. Nowadays, the nonlinear dynamics approached has been widely used in the domain of EEG analysis. According to the nonlinear dynamics theory, EEGs are nonlinear time series, which are produced by the brain and exhibit chaotic behavior. Nonlinear parameters could measure and qualify the complexity of the brain activity, such as the correlation dimension, Kolmogorov entropy, Lyapunov exponent and so on.

In this paper, we will view how the nonlinear dynamics theories are applied to the analysis of EEG time series, and we hope to present a deeper explanation of the cerebral function through these nonlinear parameters.

## 2 Nonlinear Dynamics Parameter

### 2.1 The correlation dimension ( $D_2$ )

$D_2$  is expressed as the following:

$$D_2 = \frac{\ln C_n(r)}{\ln r}, \text{ where } C_n(r) = \frac{1}{N(N-1)} \sum_{i,j=1, i \neq j}^N \theta(r - |\zeta_i - \zeta_j|)$$

Evaluating the complexity of the system, it is often calculated using the Grassberger and Procaccia (GP) algorithm with time-delay embedding.<sup>[1]</sup>

### 2.2 Kolmogorov Entropy ( $K_2$ )

Kolmogorov entropy estimates the behavior of the trajectories over time. Higher positive  $K_2$  suggests chaos. Its expression is as follows:

$$K_2 = \frac{1}{\Delta t} \log_2 \frac{C_d(r)}{C_{d+1}(r)}$$

### 2.3 Lyapunov exponent ( $L_1$ )

Lyapunov exponent estimates the rate of the divergence of trajectories of the attractor.

$$L_1 = \frac{1}{t_k - t_0} \sum_{k=1}^t \log_2 \frac{L'(t_k)}{L(t_{k-1})}$$

## 3 The Application of Nonlinear Dynamics in the EEG Analysis

### 3.1 Sleep Stage

Sleep stages and sustained fluctuations of autonomic functions, such as temperature, blood pressure, electroencephalogram (EEG), etc., can be described as a chaotic process.

In the 1968, Rechtschaffen and Kales scored the sleep-EEG and applied an algorithm, proposed by Grassberger and Procaccia (1983) to compute the correlation dimension of different sleep stages. In 1993, Fell, J.; Roehke, J., etc.<sup>[2]</sup>, calculated  $L_1$  for sleep EEG segments of 15 healthy men corresponding to the sleep stages I, II, III, IV, and REM. And they found statistically significant differences between the values of  $L_1$  for different sleep stages. U, R.A.; Faust, O.<sup>[3]</sup> analyzed the sleep data by using non-linear parameters: correlation dimension, fractal dimension, largest Lyapunov entropy, approximate entropy, Hurst exponent, phase space plot and recurrence plots. These non-linear parameters quantify the cortical function at different sleep stages and the results are tabulated.

The nonlinear dynamics methods to EEG are also used to monitor the depth of sleep. Liu, Hui<sup>[4]</sup> used the Approximate entropy, Lempel-Ziv complexity and power spectral entropy (PSE) to measure EEG time-series complexity. For 8 healthy volunteers without any medication, analysis of EEG using above mentioned methods is performed. Results show that the PSE of EEG signals during sleep process can correctly affect sleep deepness, and accord with the results of sleep stage by expert.

### 3.2 Anesthesia

Since the fluctuations of human sense could be described as a nonlinear process, researchers have began to apply chaos theory to monitor the depth of anesthesia.

Widman, G., etc.<sup>[5]</sup> computed a phase space based nonlinear correlation index from consecutive EEG epochs of 17 patients anesthetized with sevoflurane. In most patients, the highest correlation is observed for the nonlinear correlation index  $D^*$ , which is found to decrease monotonically with increasing (estimated) depth of anesthesia,

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even when a "burst - suppression" pattern occurs in the EEG. The findings show the potential for applications of concepts derived from the theory of nonlinear dynamics, even if little can be assumed about the process under investigation. Xu - Sheng Zhang, etc.,<sup>[6]</sup> presented a new approach for quantifying the relationship between brain activity patterns and depth of anesthesia (DOA) by analyzing the spatio - temporal patterns in the EEGs using Lempel - Ziv complexity analysis. Twenty - seven patients undergoing vascular surgery were studied under general anesthesia with sevoflurane, isoflurane, propofol, or desflurane. Complexity of the EEG was quantitatively estimated by the measure  $C(n)$ . The results demonstrate that in the asleep - state, the brain activity is depressed by the anesthetic and results in a loss of complexity.

### 3.3 Process of Cognition

Cognition process is directly related to the brain functionality and is a dynamically changing system. In the course of cognition process, the activity complexity of the neuronal units is continually shifting. This phenomenon can be viewed with nonlinear EEG measures.

Hao Yang, etc.,<sup>[8]</sup> observed the dynamical difference between EEG changes during mental tasks using the following nonlinear dynamics measures: correlation dimension ( $D_2$ ), point correlation dimension ( $PD_2$ ), Kolmogorov entropy and Lyapunov exponent  $L_1$ . The conclusion is as following: when eyes are closed and resting, not much mental activity goes on. This is reflected in the topographic map in which  $D_2$  levels only slightly change along time around the whole brain. This is not the case during the vocabulary association, when  $D_2$  levels of T3, T4, T5, and T6 quickly increase and spread toward both sides of the brain. This indicates that vocabulary association activity is a complex cognitive process.

### 3.4 Schizophrenia

Nonlinear methods are now beginning to be applied in the domain of psychiatric neuroscience. These applications are relevant from the level of complex genetic architecture and calcium channel dynamics to the symptomatic, behavioral, and functional outcome of schizophrenia. Analyzing these patterns of molecules to molar phenomena via nonlinear systems methods can provide new approaches to understanding complex temporal and sequential shifts in neural substrate activity, pathophysiology, and the course and treatment and outcome of schizophrenia.<sup>[9]</sup> Lee, Y. - J. etc.<sup>[10]</sup> applied nonlinear dynamics theory to EEG analysis of schizophrenic patients and estimated the correlation dimension with both temporal embedding and spatial embedding. They concluded that more complex activity occurred in certain lobes of schizophrenic patients and a relative lower global correlation dimension was obtained. This shows that there might be a diffuse slow wave activity through a schizophrenic's global cerebrum. Roberto Homero,

etc.,<sup>[11]</sup> analyzed the EEGs of 12 schizophrenic patients and 12 control subjects by means of methods from nonlinear dynamical system theory. From the EEG data, they reconstructed the chaotic dynamic attractors expressed in the phase space and calculated their correlation dimensions ( $D_2$ ). Their results showed a diminished complexity in the EEGs of the patients. This conclusion also confirms findings associated with the fact that schizophrenic patients are characterized by less complex neurobehavioral measures than normal subjects.<sup>[12]</sup>

### 3.5 Dementia

Alzheimer's disease (AD) is the main cause of dementia in western countries. Although lots of efforts have been made to develop treatments such as the Acetylcholinesterase inhibitors, which are claimed to slow the progress of the diseases, unless a sufferer is diagnosed in the early stages, can the treatments give the maximum benefits.<sup>[13]</sup> Potentially, analysis of EEG could provide the basis of a practical method for early detection of dementia.<sup>[14]</sup>

Nowadays, many researchers focus on the methods of nonlinear dynamics. G. T. Henderson, etc.,<sup>[15]</sup> developed four fractal dimension measures, and applied them to 21 EEG recordings. They concluded that the subject specific variability of the fractal dimension is an important candidate method for identifying patients with dementia. Abasolo, D., etc.,<sup>[16]</sup> applied approximate entropy (ApEn) to analyze the EEG background activity of patients with a clinical diagnosis of Alzheimer's disease and control subjects. They have divided the EEG data into frames to calculate their ApEn. The results show that the degree of complexity of EEGs from control subjects is higher. Applying the ANOVA test, they have verified that there was a significant difference ( $p < 0.05$ ) between the EEGs of these groups.

### 3.6 Epilepsy

Many researches about the seizure of epilepsy based on the nonlinear dynamics have been done. Yambe, T. etc.<sup>[17]</sup> evaluated the EEG of patients with epilepsy based on the chaos analysis. The chaos attractor was reconstructed in the phase space and the correlation dimension. KS entropy calculated from the Lyapunov exponents was evaluated. Before the seizure attack, the KS entropy showed a lower value when compared with the time series data recorded during healthy condition. This result suggests that it is possible to predict the seizure attack by the chaos analysis of the EEG signal. Winterhalder, M., etc.<sup>[18]</sup> suggested standards to assess seizure prediction performance of time series analysis techniques, and presented assessment methods originating from nonlinear dynamics with respect to their ability in predicting epileptic seizures.

## 4 Future Work

Nonlinear dynamics will be a powerfully fruitful avenue for un-

derstanding brain operations and higher functions. Through the nonlinear systems methods, researchers present more information to understand complex cortical dynamics, and explore more practical treatment to the cortical diseases.

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## 非线性动力学在脑电信号分析中的应用

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**摘要:** EEG 是由大脑产生的非线性时间序列, 体现出混沌行为。近年来迅速发展的非线性动力学理论为脑电信号分析开创了一个新的领域。本文综述了近年来非线性动力学在脑电信号研究中(睡眠阶段, 麻醉深度, 认知过程, 精神分裂, 痴呆及癫痫)的进展, 以期对脑神经动力学有更好的理解。

**关键词:** 脑电信号; 非线性动力学; 睡眠阶段; 麻醉深度; 认知过程; 精神分裂; 痴呆; 癫痫

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